A High-Order Discontinuous Galerkin Approach for Physics-Based Thermospheric Modeling

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1. Motivation

- Big interest towards the exploitation of the Low-Earth Orbiting (LEO) region of the atmosphere ⇒ mega-constellations of satellites.
- Need for accurate trajectory prediction in orbits heavily influenced by **atmospheric drag**.
- **Physics-based models** of the ionosphere-thermosphere to improve forecasting capabilities of empirical models.

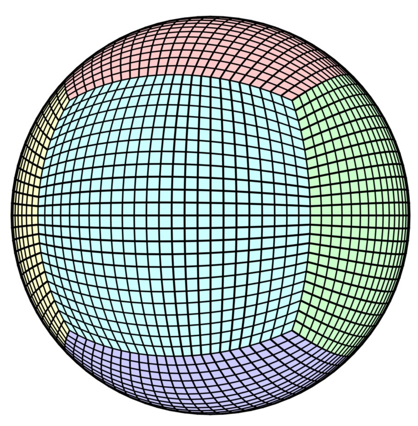
➢ Introduce modern **high-order numerical methods** for physics-based thermospheric modeling [1].

- High accuracy at reduced computational cost.
- Ability to resolve complex physics.
  - Multiscale variations, coupled phenomena....

2. Computational Approach

Exasim

github:exapde/Exasim publication:SoftwareX



- High-order discontinuous Galerkin (DG) code [2].
- Implicit-in-time: DIRK temporal schemes.
- Matrix-free approach, optimized for **GPUs**.
- Automatic code generation.

Ability to **preserve accuracy** in adapted and unstructured meshes.

- Mesh refinement to handle complex physics.
- Cube-sphere meshes to avoid pole singularities.

3. Physics-Based Model

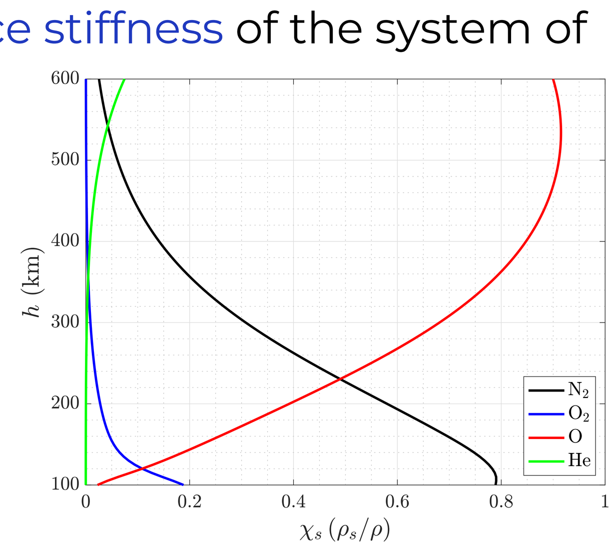
Physics-based thermospheric model consisting of a neutral gas mixture in non-hydrostatic equilibrium.

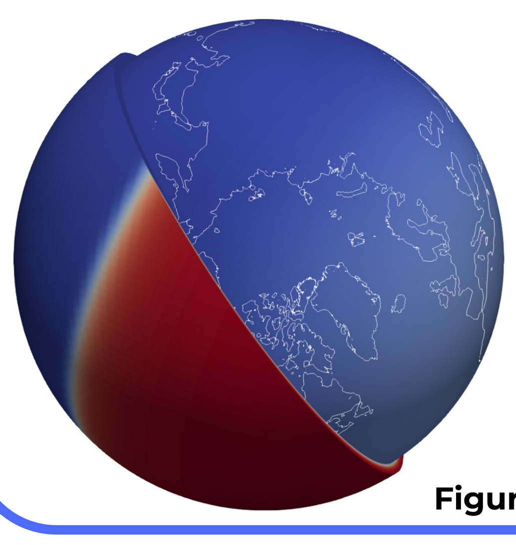
- Compressible Navier-Stokes equations in the rotating Earth.
- Non-conservative formulation to handle multiscale variations.

$$\boldsymbol{u} = \begin{bmatrix} \log(\rho) \\ \sqrt{\rho} v_i \\ \sqrt{\rho} T \end{bmatrix}$$

➢ Ensure positivity and deal with low densities.
➢ Scaled variables to reduce stiffness of the system of equations.

- Neutral gas mixture with chemically frozen composition.
  - ➢ A priori knowledge of composition of most relevant species.
  - ➢ Variable properties with altitude.





- Heating source due to **EUV radiation**.
  - ➢ Measures amount of energy being transferred due to photoionization.
  - ➢ Based on EUVAC model [3], driven by the $F_{10.7}$ index and empirical and measured data of the EUV spectrum.

Figure 1: EUV heating source distribution on the globe at a given time.

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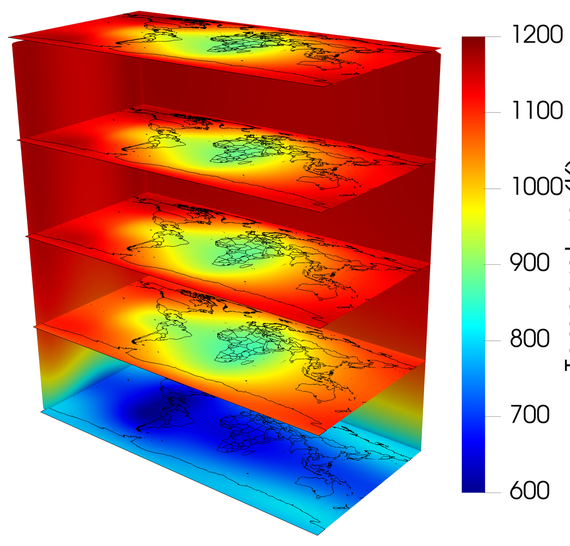
Simulation results for GITM [6] and TIE-GCM [7] have been obtained through the Community Coordinated Modeling Center (CCMC) simulation services at NASA Goddard Space Flight Center.

4. Computational Study

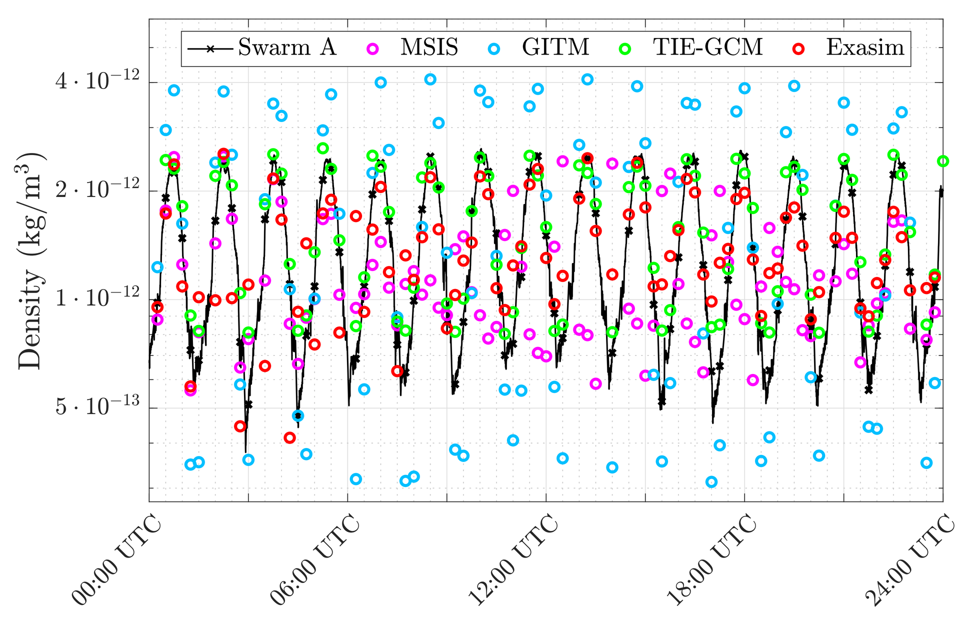
Characterization of thermospheric outputs for 2014 spring equinox.



- Quadratic approximation on a cube-sphere mesh with nearly 220k elements (30M dofs).
- Coherent patterns of density, temperature and velocity.
- Symmetric solution with respect to the Equator line, consistent with solar equinox conditions.

Figure 2: Density estimates at different altitude levels (150, 250, 350, 450 and 550km) at March 22 at midnight (UTC).

Validation with Swarm A satellite data



	MSIS	GITM	TIE-GCM	Exasim
RMS error	0.5325	0.4871	0.1531	0.2528

Figure 3: Density along the Swarm A trajectory (top) and RMS error with respect to accelerometer data [5] (bottom) for Exasim, MSIS [4], GITM [6] and TIE-GCM [7].

1D-3D validation

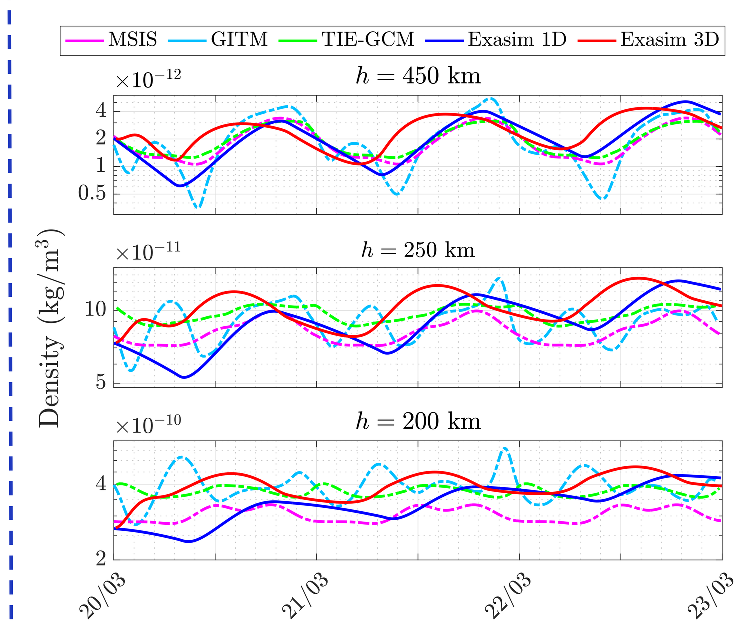


Figure 4: Comparison of the density temporal evolution at different altitudes above Jicamarca, Perú for different thermospheric models during March 2014.

- Error within common ranges for physics-based thermospheric models.
- Good agreement between 1D and 3D models.

5. Conclusions

High-order DG method for space weather modeling.

- Implicit solver with automatic code generation.
- Matrix-free approach, optimized for GPU computing.

Thermospheric model: neutral gas mixture under EUV heating.

- Scaling of variables for multiscale variations and stiffness.
- Close reproduction of dynamical behavior of the thermosphere.

Ongoing work

- Ionospheric model development (charged particles, chemistry).
- Integration with uncertainty quantification and data assimilation.

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